Risk Stabilization and Asset Allocation*

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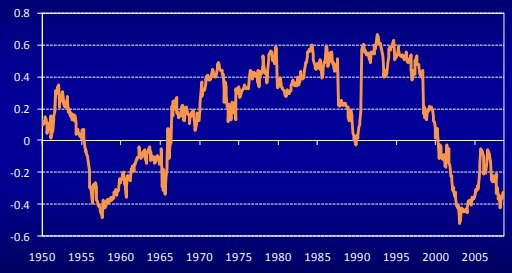
*Based on "Risk Stabilization and Asset Allocation" by André F. Perold, manuscript, March 2008

Purpose of Asset Allocation

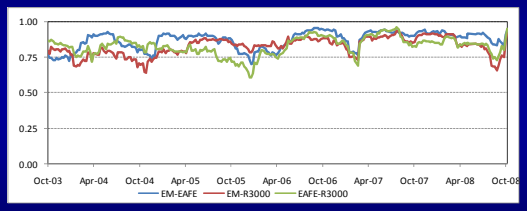
- To make efficient tradeoffs between broad asset class risks and expected returns
- These tradeoffs usually are expressed through means of a "policy portfolio", e.g. 60/40 equities/bonds
- Policy portfolios are static asset allocations

Stock Market Volatility (VIX)

The Changing Stock-Bond Correlation



Global Equity Market Correlation



Time Varying Risk

- When asset class risks and correlations are time varying, the risk of a static allocation is also time varying
- A static allocation thus is a very noisy representation of portfolio risk

Stable Risk Policies

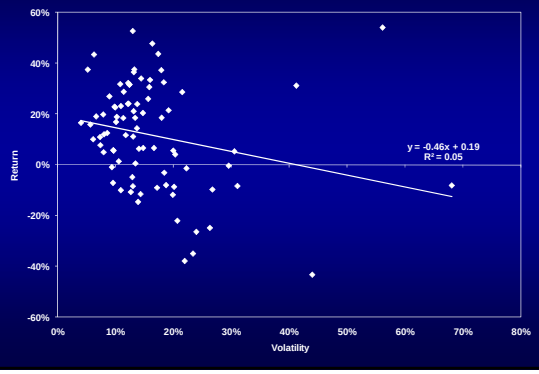
- Vary the asset allocation to maintain stable portfolio risk
- Example:
  - Reduce the exposure to global equities when equity market risk is high
  - Increase the exposure to global equities when equity market risk is low
- Basic questions:
  - Is risk forecastable?
  - What is the relationship between risk and expected return?

Theoretical Relationship Between Expected Return and Risk

- Optimal exposure
  = Risk Tolerance x Risk Premium/Variance
- In equilibrium, exposure = constant
- Theoretically, therefore
  Risk premium = proportional to variance

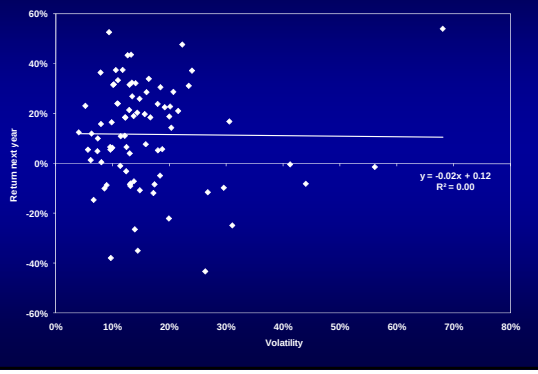
U.S. Equities: Risk vs Contemp. Return

(Intra-Year Volatility, 1926-2008)



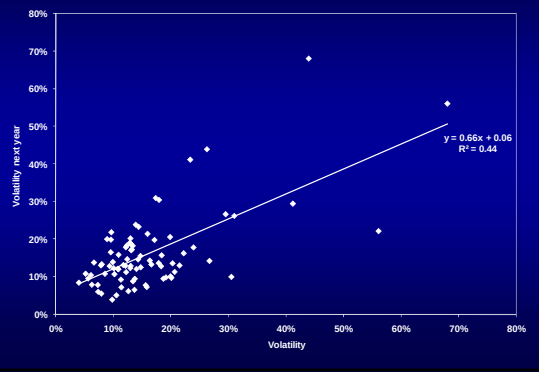
U.S. Equities: Risk vs Future Return

(Intra-Year Volatility, 1926-2007)



Persistence in Volatility

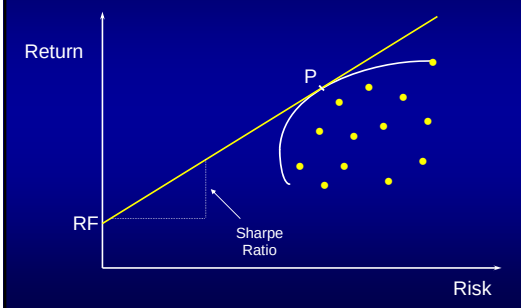
(Intra-Year Volatility, 1926-2008)



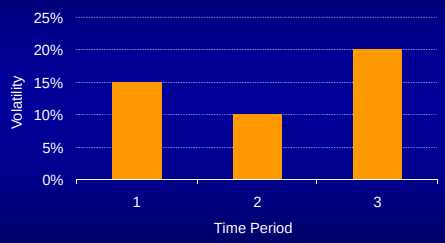
Stylized Facts

- Volatility varies significantly
- Volatility is predictable
- Volatility seems only weakly related to expected return

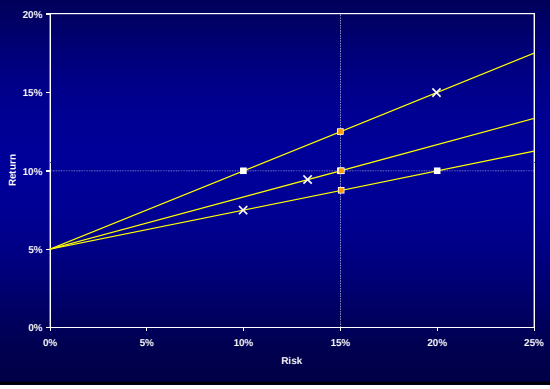
Efficient Frontier



Changing Volatility: An Example



Changing Volatility (cont.)



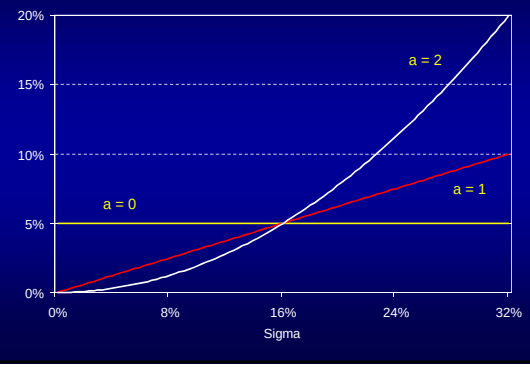
Application to Active Management

- Actively managed fund
  = Index + long/short portfolio
- Market neutral hedge fund
  = Cash + long/short portfolio
- Example: 40 best buys and 70 best shorts
  $long = $short
- How much exposure to the long/short portfolio?

The Model

- Allocation x_t to a risky portfolio and $1-x_t$ to a riskless asset
- Risk Premium is proportional to σ_t^a
  - a = 2: Proportional to variance
  - a = 1: Proportional to standard deviation
  - a = 0: No relation to risk

Expected Return vs. Sigma



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- Log σ_t is normally distributed with variance = v^2
- σ'_t is a forecast of σ_t with correlation b

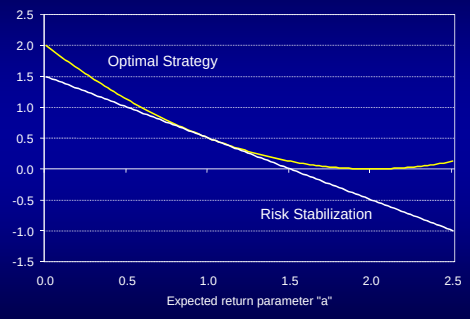
Strategies

- Static policy allocation
  - X_t = constant
- Stable risk policy
  - $X_t = \sigma^*/\sigma'_t$
- Optimal strategy
  - $X_t = \text{constant}/\sigma'^{(2-a)}_t$

Main Finding

Policy	Sharpe Ratio
Stable Allocation	S
Optimal Strategy	$S \times \exp \frac{1}{2}(a-2)^2b^2v^2$
Stable Risk	$S \times \exp (3/2-a)b^2v^2$

Sharpe Ratio Improvement



Economic Significance

v	b	Static Allocation Sharpe Ratio	Sigma
0.5	0.8	0.5	10%

	Increase in Sharpe Ratio		Increase in Portfolio Expected Return	
a	Optimal Strategy	Stable Risk	Optimal Strategy	Stable Risk
0.0	0.38	0.27	1.89%	1.36%
0.5	0.20	0.17	0.99%	0.87%
1.0	0.08	0.08	0.42%	0.42%
1.5	0.02	0.00	0.10%	0.00%
2.0	0.00	-0.08	0.00%	-0.38%

Investing when risk varies over time

- Static Policy Allocation
  - Random risk
  - Lowest Sharpe Ratio (if returns vary slowly with risk)
- Stable Risk Policy
  - Random exposure
  - Average Sharpe Ratio
  - Don't need to know expected return
- Optimal Strategy
  - Random risk and exposure
  - Highest Sharpe Ratio
  - Need to know expected return